Lifelong Learning Programme

Multiculturalism, Migration, Mathematics Education and Language
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INTRODUCTION TO AN ANCIENT MAGIC SQUARE

by Marie-Hélène Le Yaouanq* and Brigitte Marin*

INTRODUCTION

The aim of this unit is to have the students work at the same time on decimal numeration and on the use of the French language, in writing and speaking in mathematics both in terms of vocabulary and explaining one's reasoning. It is also to allow verbal exchanges about written and oral numeration used yesterday and today in various countries and to highlight the input of other civilizations to the construction of mathematics in Europe.

This unit is based on this ancient magic square discovered in 1956.

http://home.nordnet.fr/~ajuhel/Grenier/car_mag.html

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Main piloting

by Marie-Hélène Le Yaouanq and Brigitte Marin

Presentation of the piloting with teachers

The first step of the project took place during an in-service teachers training aiming at using the history of mathematics in class. The document representing the ancient magic square was given as such to the teachers. They had to discover the numbers in their current writing hidden behind the symbols.

As most of the teachers did not know the Arabic numerals appearing in the square, they had to look for a strategy to decrypt it. Three teachers whose mother tongue was Arabic directly transcribed the square and found that it was not magical! They were asked to set up a different approach. They rather quickly gave up trying with the algebraic approach and they set up reasoning on units and tens and on the number of time they appeared.

Once the square was decrypted, the trainers gave information about the used symbols, which were the ancestors of the current Arabic numerals. Some figures writing had changed so it led some teachers to be wrong. Then the trainers asked the teachers to find a use for this document in class.

So this first part had a double goal for the trainers: on the one hand to put the teachers in a search situation which they would have to reproduce with their own students in class, exchanging with teachers on their practices, and on the other hand to make the teachers aware of the search difficulties on the given document to enable them to adapt the situation.

A priori analysis

1. Mathematics concepts involved and curricula

The students' activities rely on numeration and on additive connections. During elementary schooling, work is done on decimal writing, first on whole numbers and then on decimal fractions, but numerous difficulties still remain in Year 6.In the Year 6curriculum, the students are explicitly expected to be able "to know and to use the value of figures according to their place in the writing of a whole or decimal number".

So the integration of the proposed work in Year 6 is normal.

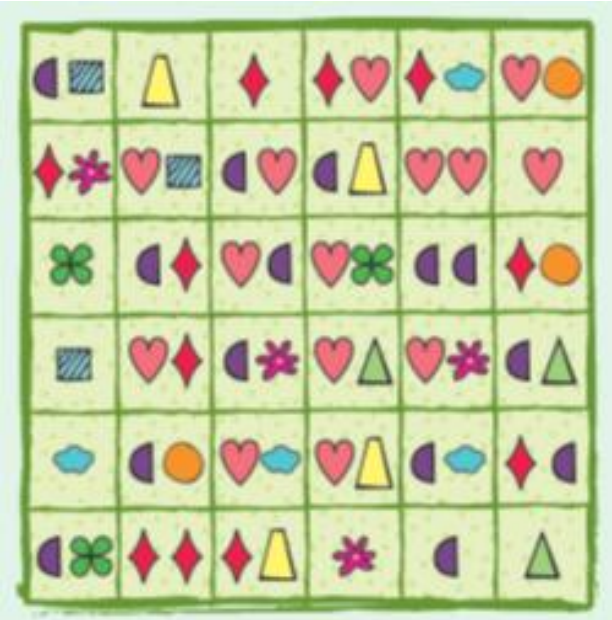
This work also relies on the taste for searching, on the ability to hold a series of reasoning steps of potentially different natures, and to use orders of magnitudes.

2. Predictable difficulties and suggested adaptations

The teachers who studied the initial magic square pointed out some difficulties:

-The difficulty to name the symbols appearing in the square, which makes the slightest communication of reasoning difficult. Then they suggested that for the

students the ancient Arabic numerals should be replaced by drawings of mathematical figures or daily life objects. Thus mathematical and daily life terms will be required to name them, so the student will have to practice mathematical or common vocabulary.



Adapted square (source: Hélice, 6th, Didier; see Annexe 1)

The difficulty to make sure that the students really understand what a magic square is, led to provide a first work step to discover magic squares with smaller sizes (3×3 or 4×4).

- It will be necessary to have a thorough comprehension of some terms that will be needed, such as figures, numbers, lines, columns, diagonals, addition and total. The work done in the discovery of the magic square will allow to introduce or reactivate these words.
- The search of the magic constant of the 6x6 square implies to add successive whole numbers from 1 to 36 and it was thought more relevant to give it so as not to increase difficulties at the beginning of the activity.

3. Description of the training sequence in class

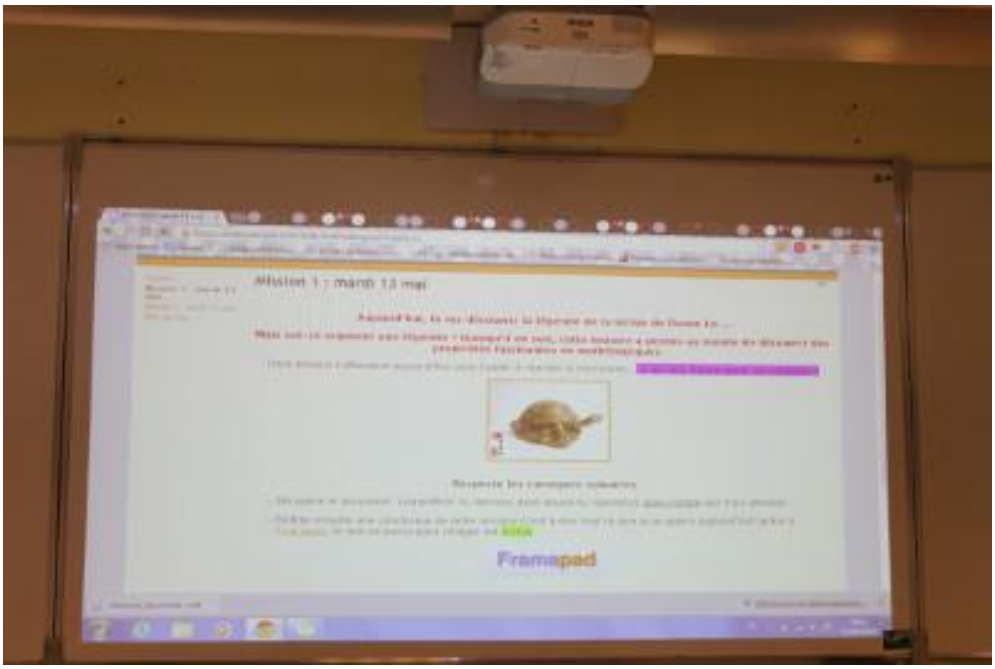
a. Predicted proceedings

The training sequence in class was adapted with the teacher who implemented it and anticipated a three -session organization, that is to say four hours.

Session 1: Magic square discovery (1 hour)

The teacher gives a mission to his students on a collaborative software (Framapad[1]): they have to discover a legend, the one of the Lo river tortoise.

[1] http://framapad.org/



Each student has a laptop connected to the Internet and launches research on the legend.

They have to discover what is hidden behind number 15, a crucial element in the legend, which is nothing but the constant of a 3x3 magic square. They then have to check that a given square is magical and to fill in three squares in order to make them magical.

To conclude, they must write a summary of what they have learnt on the collaborative tool. The session is filmed.

Session 2: Group work on the adapted square (2h)

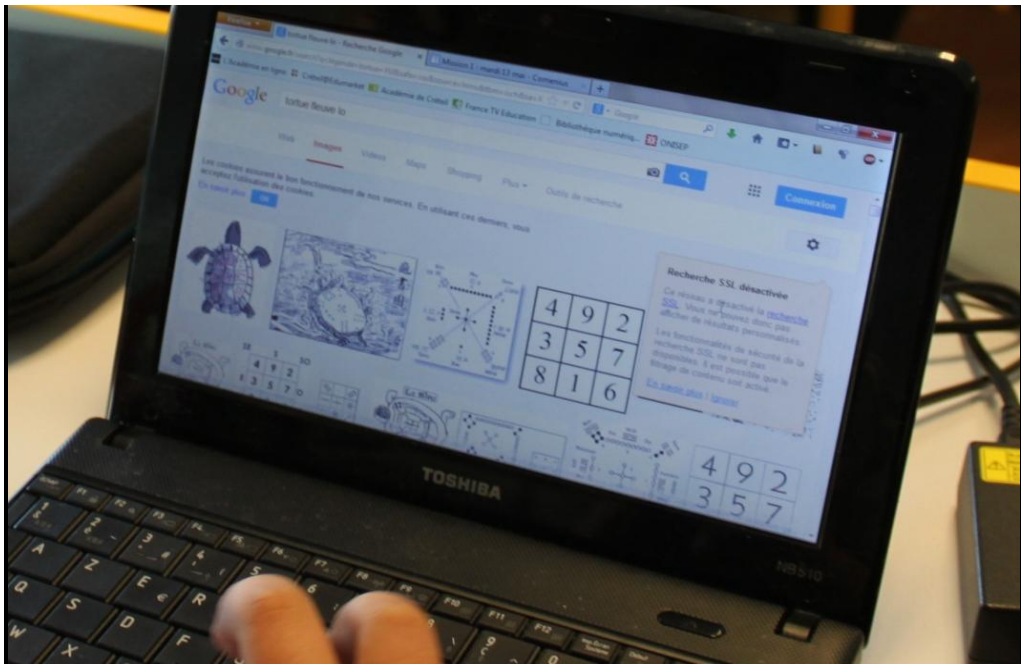
It's a two consecutive hour session. The adapted square is handed out to the students, with questions enabling them to decrypt it. The students work in groups of three or four in order to promote exchanges and reasoning formulation. The session is filmed and dialogues between the students are recorded.

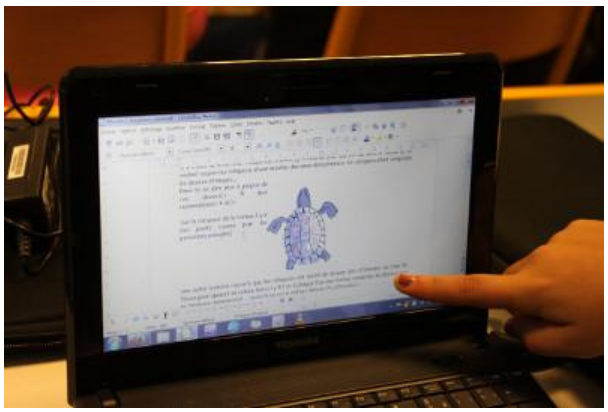
Session 3: *Collective summary* (1h)

This session will be dedicated to the review of the work done, to the study of the initial square, to exchanges and to cultural and historical inputs.

b. Real proceedings and analysis

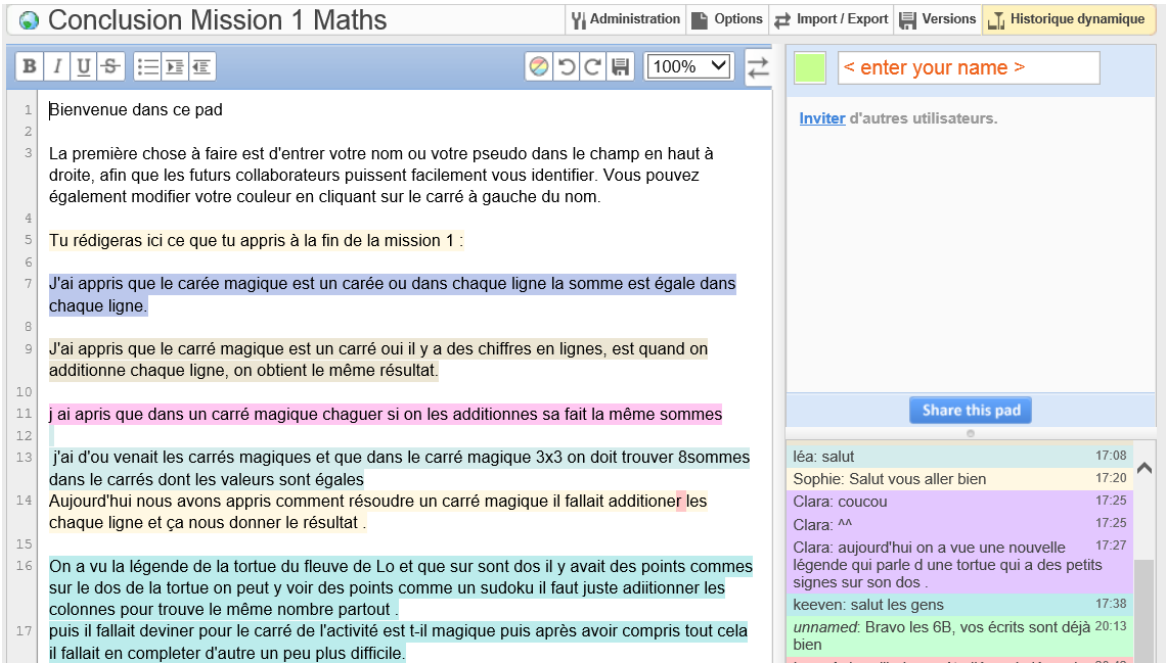
Session 1: The "mission" aspect highly motivated the students during the first session. Big differences in the mastering of the online search for information could be observed, for example in using or not keywords in a search engine, and then while making a selection among given websites, in choosing to ask for text or pictures, between rewriting or copying and pasting to answer questions in the teacher's document



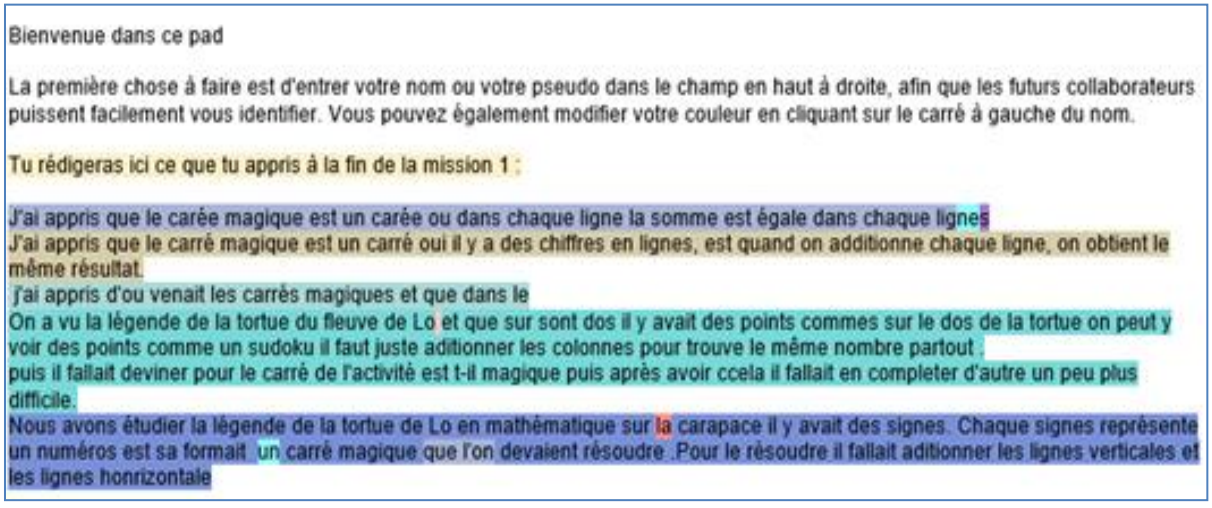


The skills to search and select the requested information, then to work together thanks to collaborative tools are part of the competences that must be worked on in lower secondary school.

The fastest students started the collective writing of the summary at the end of the session, and the other ones finished it at home the day after. One could notice that the fastest students only based their summary on the magic square, whereas the students who encountered more difficulties during the session, remained on the legend and dedicated their summary to a description of the tortoise.



Some students reread and corrected mistakes in sentences that had already been written by others (change of colour in the line). Some activities of writing, of reading and correcting, and of reviewing of transitional writings are at stake, on a differentiated basis, during a short time, and well included in the work in mathematics.



They write a final summary of what they have learnt on the collaborative tool, with the teacher's help.

The students newly arrived in France only had to write a sentence on a piece of paper describing what they had seen (the tortoise, the points…).

Session 2

Session 2 will begin with a review of the collective summary on the collaborative tool written by some students between the two sessions, in order to simplify andcomplete it.

Then the search on the adapted square can start.(Annexe 1, questions 2, 3, 4)

An important difficulty appears at the beginning of the activity: the students understand well that each symbol hides a figure, but for some of them the ten symbols consequently represent 1,2,3,4,5,6,7,8,9,10.

But interpreting that this square contains consecutive whole numbers from 1 also represents a problem, because, for most of the students, numbers start from 10 or even 11. It is an unexpected difficulty: for these students, 0,1,2,3,4,5,6,7,8,9 are not numbers but digits.

Another difficulty consists in differentiating the number of occurrences of a symbol from the figure it represents. Discovering the first two figures takes a long time, searching is easier afterwards but four symbols are still to be decrypted at the end of the session.

Session 3

The search is soon over. Some pupils realize that one diagonal was not used in the reasoning and ask to check the total of the numbers written in this diagonal. The initial square is then given to the students as well as explanation on its discovery in China. A student makes analogy with the decrypted square.

Another student newly arrived in France reads and writes some of the Arabic numerals that he recognizes, which generates a very strong interest in the class. The students then show a great deal of imagination to try to explain the reason why the square was found in China. Then the teacher provides information on the history of the Arabic numerals.

A posteriorianalysis

First it can be noticed that this training sequence raised the students' interest. Skills in reading, searching and selecting information have been worked on. The use of a collaborative tool enabled to have some students take part in written work in various ways: by writing, rewriting, revising, correcting...

We can also notice that some mathematical terms are not spontaneously used at the beginning of the session, such as "lines" and "columns", often replaced by "horizontal lines" and "vertical lines", but that they are correctly used at the end of the session.

However, the designation of the symbols showed a real difference between the "EANA"[2] students and the others.

2 "Elèves allophones nouvellement arrives" en France

The EANA students[3] rely on school language, using vocabulary they have learnt since their arrival (from 1 to 5 months). They quote the circle, the half circle, the rhombus, the triangle and the rectangle. French speaking students havemore recourse to daily life things (moon, apple turnover). Thus they had difficulties in naming the red shape and none of them suggested the rhombus. Finally they all mentioned the square instead of the rectangle, and therefore seemed to be using a generic word in reference to a daily concept instead of the scientific concept studied during the mathematics course. That did not cause any problem of communication between them.

The main difficulties clearly appeared in the understanding of the questions and in the explanation of arguments. Complex syntactic constructions are sometimes useful and require reformulating.

But, above all, the language used must exactly express the notions at stake. The meaning of the words "figure" and "number" varies according to the context, in the mother tongue or in mathematics, and it appeared that these notions were not fully mastered by the large majority of the students. However they performed without any difficulty isolated school tasks that were repeated and classic in the work on numeration, such as giving the tens figures of a number.

The framework of solving a problem, in communicative situation and interplay involves another level of availability of mathematical knowledge and of mastery of the language.

Bibliography, sitography:

Chabanne J.-C., Bucheton D.(2002) *Écrire en ZEP : un autre regard sur les écrits des élèves*, Delagrave édition - CRDP Versailles http://www.cndp.fr/bienlire/04-media/b-biblio03.asp?prodid=42772 (availableaddress on 2015 May 01)

Vygotski, L.S. (1934). *Pensée et Langage*, Editions sociales (Traduction de F.Sève, 1985).

Marin B. (2011-2012), La reformulation en classe: un discours équivoque, *La construction des inégalités scolaires,*sous la direction de Rochex J.-Y., Crinon J., Presses Universitaires de Rennes.

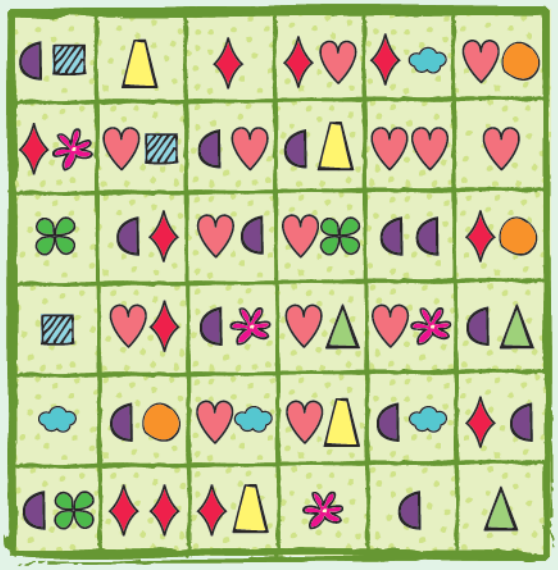
IfrahG. (1994), *Histoire universelle des chiffres*, édition Robert Laffont.

Saint-Andrews University, *The Arabic numeral system*, http://www-history.mcs.st-and.ac.uk/Indexes/Arabs.html (available address on 2015 May 01)

[3]Three students whose language is Portuguese; one student from Bangladesh; one from Pakistan and one from Sri-Lanka.

Annexe 1: (source: Hélice, 6th, Didier)

@ Un grand carré magique



Ce carré est magique mais chaque chiffre est remplacé par un symbole. Cherchons à découvrir tous les nombres contenus dans les cases !

1 Préparation du travail

a/ Reproduire ce carré avec des cases vides.

b/ Dessiner sous ce carré les symboles utilisés dans le carré magique ci-contre.

2 La constante magique

La constante magique de ce carré est 111. Expliquer ce que cela signifie.

3 Les chiffres des dizaines

a/ Ce carré contient les premiers nombres entiers à partir de 1.

Quels sont les nombres qu'il contient ? Quels sont les chiffres des dizaines possibles ?

b/ Compter le nombre d'apparitions de chaque symbole comme chiffre des dizaines et démasquer un premier chiffre.

c/ Écrire le chiffre démasqué dans le carré vide partout où il se trouve.

d/ Dans la 1re colonne, quel doit être le chiffre des dizaines inconnu pour pouvoir obtenir la constante magique ? En déduire le 3e chiffre des dizaines à démasquer. Compléter le carré.

4 Les chiffres des unités

a/ Quel symbole des chiffres des unités n'est jamais seul dans une case ? Quel chiffre cache-t-il ?

b/ Observer la dernière colonne : quel chiffre permet-elle de découvrir ?

c/ Quel chiffre la 3e ligne permet-elle de découvrir ?

d/ Quel autre chiffre peut-on alors connaître ? (Penser aux diagonales.)

e/ Finir de remplir le carré. Vérifier que le carré obtenu est bien magique !

5 Un carré bien ancien

Observer un très vieux carré magique conservé au musée de Xian en Chine, sur le site : http://home.nordnet.fr/~ajuhel/Grenier/car_mag.html. Traduire chacun des chiffres inscrits.

Second piloting

by Maria Piccione[**]

Introduction

The proposal concerns arithmetical concepts: in particular, numeration (decimal and positional writing system), additive relations and, more in general, symbolic representations. It offers an adequate context to deal with fundamental curricular contents of Lower Secondary School, as it can be found in the Italian national guidelines.

The proposal was firstly described by the trainer and discussed with the two teachers of the classrooms where it was going to be piloted.

The proposal

The work context refers to the decrypting of an old magic square discovered in China.

The activity aims at revisiting already known concepts, in order to improve their knowledge and to explicitly express certain fundamental properties, as well as at developing symbol sense. In other words, it promotes a metacognitive analysis of *sign-sign meaning* concepts in Arithmetic to clarify the relation "natural number - symbolic representation", and therefore the relation "digit-position-value" and finally the "order of magnitude" concept. Hence it faces the cognitive obstacle corresponding to the epistemological obstacle when passing from number concept to numeral concept.

In addition, the proposal gives rise to significant opportunities for introducing both the demonstration activity and the algebraic thought, and for thinking on geometrical questions. The learning context is suitable to promote the development of language skills in understanding and writing a text and in discussing by explaining plans-strategies-solutions.

As regards the *affect* domain, the approach uses two methodologies, game and narrative, which are not only useful to stimulate curiosity, fantasy, creativity, discovery, role assumption but also to prevent states of anxiety, frustration or feelings of inadequacy.

A historical and multicultural perspective can be used to outline the developments of written and oral numeration from the past to current days, in various countries and cultures.

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In particular, it allows to show the gap between the instinctive human activity of "counting" and the slow process which brought to elaborate a writing system using a few signs to represent even large numbers.

The piloting

The teaching unit was piloted in two classes of the second year (7th grade) of the Lower Secondary School "G. Papini" (Castelnuovo Berardenga, Siena). It involved 42 pupils (11 immigrant and 10 with cognitive problems) and two teachers (V. La Grotteria & P. Sabatini) with the collaboration of the trainer. The activity was carried out by following the general design provided by the French project team, structured in four sessions, with two arguments in addition, the former being a geometrical digression and the latter an arithmetical scheme construction. These two steps were not scheduled in the plan, but they emerged during the class-work. They corresponded respectively to a *metacognitive-linguistic aim* and a *cognitive aim*, that is:

- to let the pupils reflect on the difficulty of explaining in technical terms even an apparently simple procedure (namely the construction of a square subdivided in congruent cells);
- to allow the pupils to construct a clear *mental image* about the way natural numbers can be arranged ten by ten, which highlights on digits recurrence in the role of units or tens from 0 to 99.

We could not use Framapad software, due to management issues of the computer lab: nevertheless this difference, with respect to the original direction, did not compromise the work from the point of view of cognitive aspects. Otherwise, it gave everybody the possibility to contribute to it in some way (at least just in the task of writing numbers).

Session 1: Approaching the magic squares (3 hours)

Presentation of the task.The work began with a short story told by the trainer (see Appendix A.1), and the display of an image of the old square:

The word "square" was investigated to verify the ability to explain the meaning of a very familiar geometrical term.

Some unexpected difficulties emerged related to the concepts of "side" and "right angle", contrarily to what resulted from the conditions of "equality between segments" and "equality between angles", which were immediately understood in terms of "overlapping".

More precisely, the word "side", which was referred, at first, to the word "boundary", gradually improved the sentences formulation referred to proper definition, according to the following steps:

- *"the part of the boundary"*
- *"the part of the boundary between two vertices"*
- *"segment joining two vertices".*

The word "right angle" caused difficulties both in formulating complete sentences and in explaining the paper folding construction as exemplified below:

- *"the space comprised between two segments which are each other …"*
- *"I fold once a part of the sheet and then I fold the other part inward, being careful not to put it either before or after";*
- *" I fold once a part of the sheet and then I fold the second crease over the first one".*

In some cases, it was related to "measure", and to "incidence between a horizontal and a vertical straight line".

Internet searching and resuming results.

By this time, the expressions "magic square"and "square of order 6" needed to be investigated. A research on the internet started in order to answer this issue.

The students, in groups of 2-3 for each computer table in the computers' lab, began searching on the Internet.

After that, each group wrote a text resuming the information they collected and deemed relevant, also relatively to the magic squares history.

Students of each group read the text aloud. Some specific terms emerged, such as "array", "order", "magic sum" and were outlined while reading; moreover some properties were underlined, in particular, the existence and uniqueness of magic squares depending on order. It generated a feeling of wonder for the big amount of them, with the surprising jump of cardinality from the third to the fourth order (from 1 to 880), and from the fourth to the fifth one (from 880 to 275.305.224),reachingthe level of the billions of billions to the next step.

Working in a collaborative way, the pieces of information gathered by the various groups were summarized together in a single text, written down by a student, with the aim of being included in a final poster exhibition.(Figure 1)

Training in a simple case. This activity was addressed to consolidate the defining properties of a magic square. Every group was given a sheet of paper containing a colored picture of a 3x3 magic square and some questions on the types of numbers

appearing in it and on the addition properties of the numbers which are satisfied in each row, column and diagonal (see Appendix A.2). As this work pointed out, the majority of the students deal with the terms "row" and "column" in the natural language sense, and with the term "diagonal" in the geometrical interpretation; whilst they do not use the word "constant", which they paraphrase as reported below:

- "*it always comes the same result*".

Some students could indicate the objects by gestures. (Figure 2)

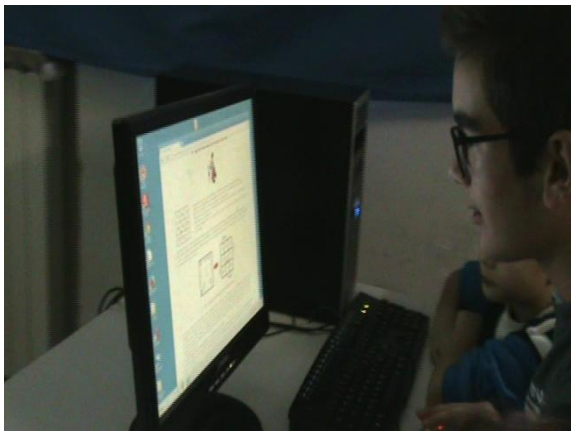

Figure 1

Figure 2

<u>Session 2</u>*: Discussing words "order of a square" and "magic sum"* (2 hours)

The students had been intrigued by the new concepts of "order of a square" and "magic sum".

Working as a whole class, with the guidance of questions posed by the teacher, the following activities was carried out to construct the order of a square concept:

- drawing on the board the square corresponding to each order (from 1 to 6)
- explaining operations to be performed
- relating the order of a square with the number of its cells
- noting the banality of the magic square of order 1 and the non-existence (impossibility) of models for the order 2.

As the attempts in verbal explanations have shown, transition from the intuitive to the rigorous idea of order has not been easy. In order to support this conceptual construction, the students performed a drawing activity, which actually resulted to be helpful: subsequently, they were able to say that the order is *"the cells number along a side"* or *"the cells number of each row", "the subdivisions number of a side".* Difficulties emerged both in realizing and in describing the steps of the sequential instruction, which denoted a low level of procedural thought development among students.

The magic sum concept has not risen problems; the discussion about it has led a (clever) student even to explicit the calculation rule of the magic sum depending on the order, arousing the enthusiasm of the classmates:

"*by the order I can find the total number of the cells; then I have to add all the numbers from 1 until this found number and finally I have to divide this new one by the rows number"*.

Recalling current numeration. Still working as a whole, the class recalled the properties of current numeration, and they were asked to explain the sense of digit and the sense of digit depending on position.

A useful instrument to perform the work has been the drawing on the blackboard of a table as in the figure below:

0	10	20	30					
1	11	21	31					
2	12	22	32					
3	13	23	33					
4	14	24	34					
5	15	25	35					
6	16	26	36					
7	17	27	…					
8	18	28						
9	19	29						

This scheme let the pupils able to get a visualization of the regular distribution of unit digits and tens digits in the written succession and to discuss about the observed occurrences.

Session :. Decrypting the magic square (1 hour)

Solving a pretty magic square**.** Once gathered in small groups, the pupils were committed to the solution of a magic square of order 6 where symbols of the original old square were replaced by other familiar figures.

They began by attempts to try the coherence of supposed numerical value of some figures; then they proceeded in a more systematic way following a few hints given by the teacher and using the "guide scheme" extended until number 36. They could carry out the strategy of counting both digits and figures recurrences and could

compare the results in order to get conclusions (first discovering the symbol of number 3, then the one of 0 …). (figure 4a and 4b).

Figure 4a

Figure 4b

<u>Decrypting the magic square of Xian</u>**.** During this session, the teacher provided each groups with a copy of the ancient Chinese magic square with the task to decrypt the Arabic numerals available. The correspondence between the symbols of the two squares was immediate. (figure 5).

Figure 5

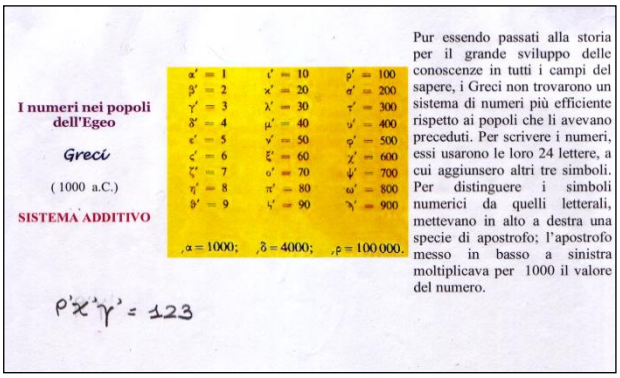

Figure 6

<u>Session 4</u>: Come back to cultural angle (1 hour)

An historical view about the evolution of Arabic numerals was done through reading and summarizing in class group-work nine tables prepared by the teachers. The material referred to numeration systems in Sumerian, Babylonian, Egyptian, Indo-Arabian, Mayan, Greek, Chinese and Roman cultures.

Two additional tables were concerning the "Numbers of computer" and the "History of zero".

Each group tried to write a number in the given system.

<u>Final test</u>. (1 hour)

A final questionnaire (see Appendix A.3) was submitted to the students to control the traces of the work done, about five months after the end of the task. The answers

provided many educational guides. The most important and general guidelines refer to the density and depth of the proposal contents which requires *time* for performing, in order to use its full learning potential.

A posteriori analysis.

The proposal appeared to be productive in gaining interest under various aspects, both on methodological and on topic related levels: historical approach, cooperative working, used tools, role interpretation, discovering and decrypting activity.

In *affect domain*, the experience, in spite of mental application, can be lived as a game, contextually to a feeling of good competition among groups. The new problem of decoding results generates moments of surprise and satisfaction, as for finding the solution by reasoning and logic efforts.

In *cognitive domain,* the activity offers a context in which the students may apply Arithmetical concepts and operations, revisiting them and increasing awareness of numeric properties and regularities, getting general formulas (the first n natural numbers sum, the "magic key" value) and improving the symbol sense.

The work requires and develops ability of

- getting information in a text in order to re-writing a text
- understanding the instructions
- explaining, defining, argueing.

The activity highlighted difficultiesreferable (in several cases) to a mechanical teaching/learning of Arithmetic. The exposure language in teaching seems to lead to generate linguistic stereotypes that give a "dress" to a weak conceptual body.

It revealed even an unexpected difficulty in the drawing procedure of a square divided in a given number of congruent cells: many pupils could not do it in an ordered way: by simply recalling a mental image, they could not explain the steps of the geometric construction of the structured figure.

The activity is adequate for training (pre-service and in-service) Mathematics teachers.

APPENDIX

A.1

The told story. *An archaeologist friend of ours went to work in China, she visited the Museum of Xian, where she had been intrigued by an archaeological finding: a metal plate with the image of a square pattern containing strange signs. The caption underneath reported the following phrase "Magic square of order6". Back to Italy, she asked us about the mathematical meaning of the plaque. Our amusement in solving the enigma is why we propose to you the same discovery game...*

A.2

Osservate questo quadrato con numeri nelle caselle.

Qui sotto abbiamo trascritto alcuni comandi e domande alle quali vi chiediamo di provare a rispondere:

- Quali numeri compaiono nelle caselle del quadrato?
- Addizionate i numeri di ogni riga del quadrato. Che cosa potete notare?
- Addizionate i numeri di ogni colonna del quadrato. Che cosa potete notare?
- E addizionando i numeri di ciascuna diagonale del quadrato. Che cosa trovate?

Adesso scrivete tutto quello che avete scoperto!

A.3

1. A scuola è arrivato un bambino nuovo…
2. Prova a spiegargli che cos'è un quadrato magico.
3. Digli che cosa ti ha colpito dell'attività svolta per riuscire a decifrare il quadrato magico con le figurine.
4. Il bambino non sa disegnare un quadrato diviso in caselle quadrate uguali. Raccontagli come si fa.
5. Inoltre è curioso di sapere come si scrive un numero nel nostro sistema di numerazione. Prova a spiegarglielo.

Third piloting

by Hana Moraová[***]

The teaching material was piloted in the 5th (primary) and 7th (secondary) grades in ZŠ Fr. Plamínkové, a Prague school. As there are almost no migrant pupils in the school, the material was piloted using the CLIL methodology i.e. the lessons had two objectives – mathematical and linguistic. This was meant to simulate the situation in which pupils struggle to understand the language of instruction in a mathematics lesson, as the language of instruction is not their mother tongue.

The lessons were video recorded.

The piloting in both classes had two primary objectives – to develop language skills of the pupils, introduction (revision) of key concepts in English (row, column, diagonal, add, multiply, sum, product etc.). The unit was meant to develop the pupils' receptive and productive language skills: the piloting began by conversation about magic, secret, superstition and legends, by narrating the legend of Lo-Shu. The pupils were also trying to answer the teacher's questions in English. The mathematical objective was search for the magic number, mathematical reasoning, formulation of arguments, discovery of properties of numerical operations. The teaching experiment was concluded by having the pupils work following a basic algorithm (how to make an odd order magic square).2 different classes, 7th and 5th graders, CLIL lesson, i.e. language and mathematics goals.

1st piloting: 7th grade, CLIL lesson, use of smart board – using the possibility to write on an existing picture of the turtle shell, have experience with mathematics in English lessons, language less of an obstacle

Language aims: speaking activity – discussing magic and miracles, listening – the Lo Shu legend

Mathematics goals: looking for patterns and regularities, discovering (re-discovering) properties of arithmetic operations, patterns in a square grid

Problems encountered – the pattern on the turtle in the presentation had a mistake, i.e. the numbers gained did not make a magic square

When asked to multiply each number by the same number, some pupils multiplied 1x1, 2x2, 3x3 etc., which does not make a magic number

Warm up: discussion – what are legends, what is the difference between a legend and a fairy tale, give examples of legends, do you like legends and magic?

Lead-in: Telling the legend of Lo-Shu

Main activity:

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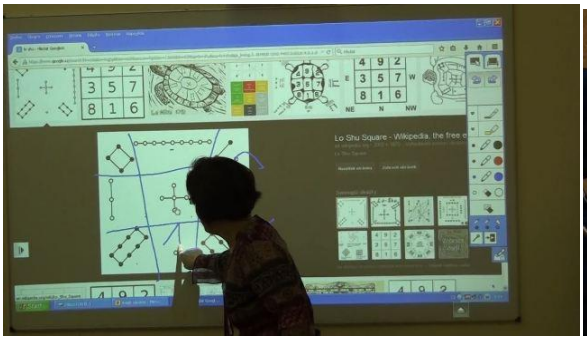
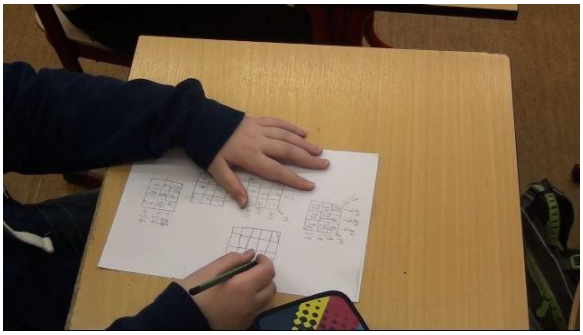
1. discovering the numbers in a magic square and what makes a magic square magic (failed because of a wrong picture)
2. discovering what happens if the same number is added to each number, will it still be magic? Why?
3. discovering what happens if each number is multiplied by the same number (misunderstood by 2 pupils)
4. discovering what happens if we swap rows or columns equidistant from the centre
5. showing the principle of putting numbers in a magic square
6. pupils work on a 5 times 5 square, trying to put the numbers, a few finish in time and successfully, the teacher monitors and tries to help if there are problems

The end of the lesson. The teacher decides to pilot the same unit again, trying to eliminate the problems from this lesson.





2nd piloting: 5th grade, CLIL lesson, use of smart board – using the possibility to write on an existing picture of the turtle shell, the first lesson of mathematics in English

Language aims: speaking activity- discussing magic and miracles, listening – the Lo Shu legend, introduction of basic mathematics vocabulary in English (odd, even, multiply, add, subtract, diagonal, line, column)

Mathematics goals: looking for patterns and regularities, discovering properties of arithmetic operations

The original lesson plan was modified to avoid some of the problems from the previous lessons (the selected image had the right pattern so the pupils could really discover the numbers on the shelf)

Warm up: What is magic? Language speaking activity, eliciting ideas from the children

Lead-in: The teacher tells the story of Lo-Shu monitoring understanding (more difficult vocabulary items like floods, turtle, sacrifice)

Main activity:

1. the picture of the pattern on the shell projected on smart board, a grid made and pupils asked to think why this is magic

   The pupils very active looking for tens of different regularities (pattern of odd, even numbers, the sum of numbers in a triangle, the sum of numbers in corners etc.), after about 7 minutes of trying out the idea od 15 discovered (through the idea that the opposite numbers have the sum of ten).

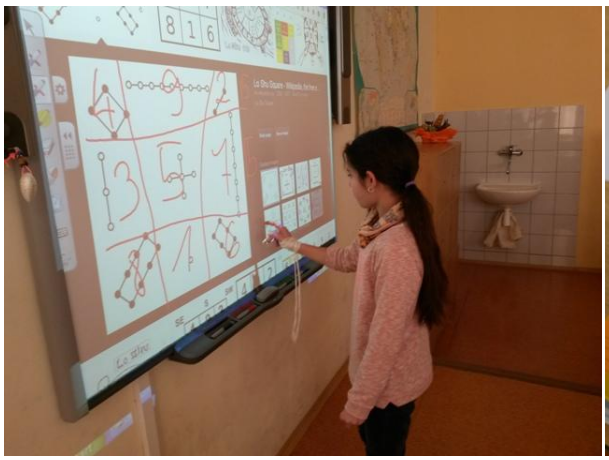
2. pupils asked to add the same number to each number, elicited one number that everybody would be working with (6); pupils work and find out that the square is still magic, the teacher asks why?

   After some proposals the pupils see that the difference between the original sum 15 and the new sum 33 is 18. With the teacher realize that 18 is three times six, e.g. the new number added three time in each row, column and diagonal.

3. pupils are asked to multiply each number with the same number (taking into account experience from 7^{th} grade number 3 elicited and first three numbers are done together as an example to avoid ambiguity]; pupils work and find out the square is still magic, the sum being 45; again asked to look for the reason why.

   They experiment, propose different ideas of why it works but need considerable help of the teacher to see that 15 times 3 is 45, i.e. that multiplying each summand, or the whole sum by the same number brings the same result.

The lesson ended. As more time was left for discovery of the magic, fewer activities were done. But the teacher evaluated this lesson as the best one and more beneficial for the pupils who had discovered more of the things on their own.



Conclusions from the three piloting

by Marie-Hélène Le Yaouanq and Brigitte Marin

These experiments carried out tend to confirm the interest of the "magic square" activity and its adaptability to different class levels (from CM2 to 5è, that is to say from the fifth year of primary school to Year 7).

First and foremost, the students are actively placed in a position of research. The presentation of the legend, the "magical" aspect of the situation, allows an involvement of the students who are receptive to the playful dimension and to a presentation that gives them the impression of taking up a challenge. This involvement even remains throughout a sequence of three or four sessions.

In France and in Italy, starting the activity by the means of a web quest followed by exchanges and by the writing of a summary, develops competences related to searching, classifying and exploitation of information, as well as competences in the mastering of either written or spoken language. In the Czech Republic, the discovery of the situation, led in English, is rather close to the discovery of the work done in France with the allophone students.

However, differences show up in the implementations.

Differences in length from 1 to 4 sessions can be observed, which are linked to different objectives in Mathematics. The mathematical work includes the discovery of the magic square but then, it focuses on numeration in the case of the decoding of the ancient magic square or the discovery of certain mathematical properties of a magic square.

The multicultural and historical aspect is managed differently in Italy, with a group work on the different numerations and in France, with live exchanges in the class, based on the presence of foreign students.

The decoding of the ancient magic square puts the emphasis on the difficulty of symbolism, but also on real reasoning difficulties concerning numeration and the position of a figure in a context that is different from "classical" exercises, which may lead to stereotyped answers without constructing the concept strongly enough. The historical aspect and the study of different numerations is consequently an interesting extension from a mathematical point of view.

The decoding of the magic square confirms that the linguistic and the cognitive aspects are intimately linked. Therefore, a difficulty in the mathematical reasoning forces the students to exchange in a precise manner in order to be able to move forward, and, at other times, difficulties to express what they have perfectly understood from a mathematical point of view make them aware of the necessity to work on the language. The case that was explored seems to provide a context allowing the development of both language and mathematical competences, while remaining motivating for the students and adaptable to different class levels and to different objectives.